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Cryptography in Blockchain Part II: Zero Knowledge Proof and ZK-SNARK

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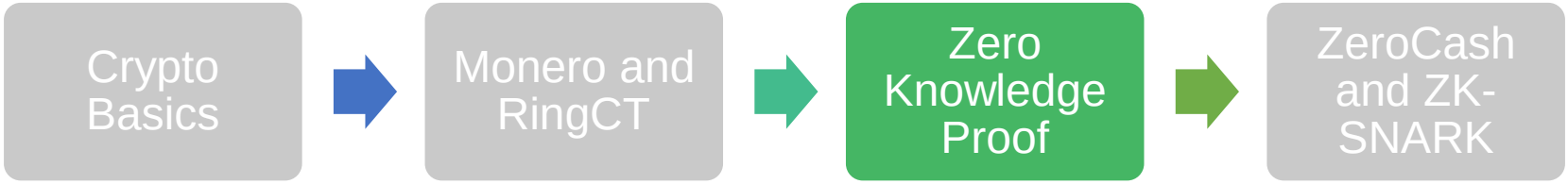
Outline


Crypto Basics
Monero and RingCT
Zero Knowledge Proof
ZeroCash and ZK-SNARK

Mathematical Proofs

- When referring to a proof in logic we usually mean:
  - 1. Based on axioms.
  - 2. A sequence of statements.
  - 3. Each statement is derived via the derivation rules.
  - 4. The proof is fixed, i.e, in any time, anyone can read it, and get convinced.
- Or, we “prove” a statement by convincing someone.
  - For example, in court the prosecutor tries to convince the judge that the defendant is guilty.
  - The prosecutor challenges the defendant.
  - In case he fails to answer in a consistent manner, we say that the prosecutor proved his point.

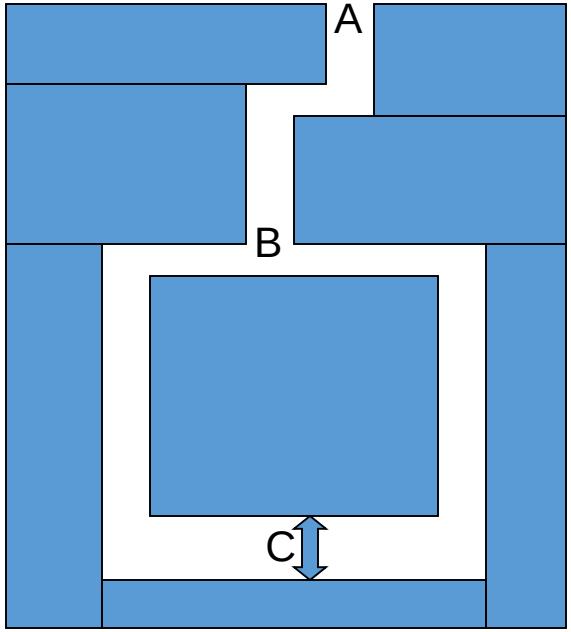
Source from Eli Biham

Zero-Knowledge Proof

- About proving somebody knows some secret without revealing it
- **Interactive:** between the ***P****rover* & ***V****erifier* (***P*** and ***V*** or ***P****eggy* & ***V****ictor*)
- Does not give away any information about the secret (zero-knowledge)
- Does not allow the *verifier* to impersonate the *prover*

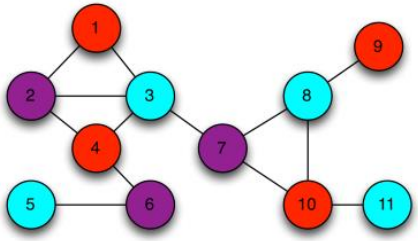
A very simple scenario

- Secret: the key to open the door (C)
- Uses *cut-and-choose* protocol
- Enough to prove to Victor
- Not enough to convince a third party (Carol) of proof's validity
- **(Implicit) Commitment: enter cannel from left or right; cannot change!!**



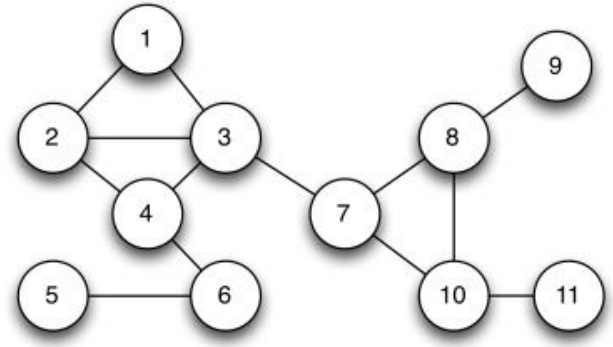
Zero Knowledge Proof Knowledge Types

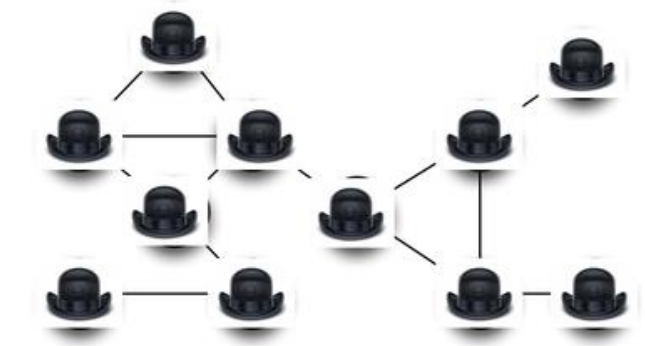
- **Statements about "facts"**
  - "a specific graph has a three coloring"
  - "some number N is in the set of composite numbers".
  - Each of these is a statement about some intrinsic property of the universe.
  - This is What

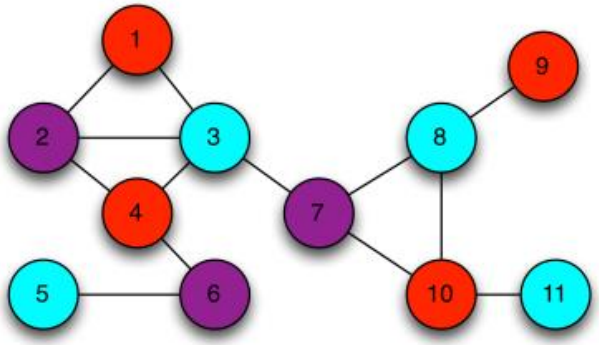


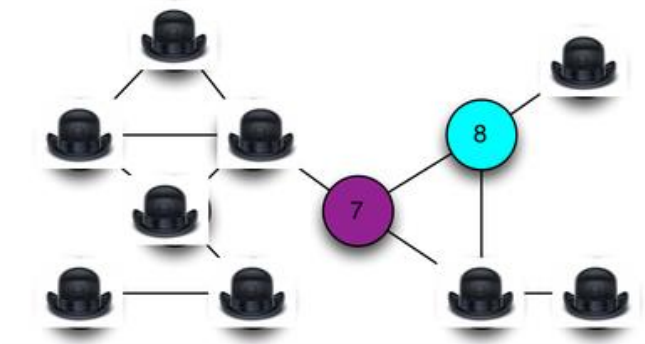
- **Statements about personal knowledge.**
  - "I know a three coloring for this graph"
  - "I know the factorization of N"
  - These go beyond merely proving that a fact is true, and actually rely on what the Prover knows.
  - This is Why

Example: 3-coloring



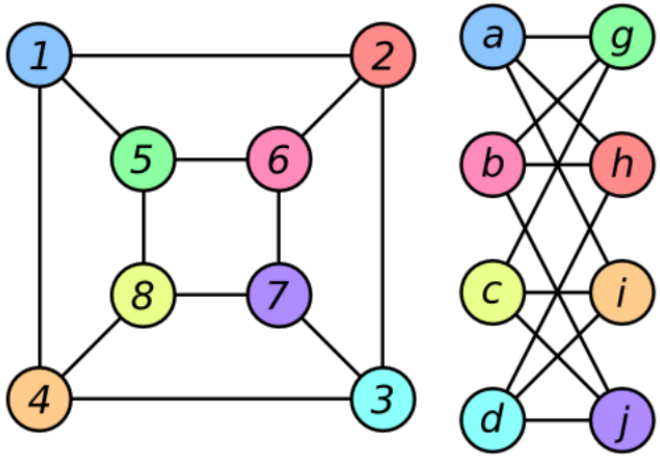






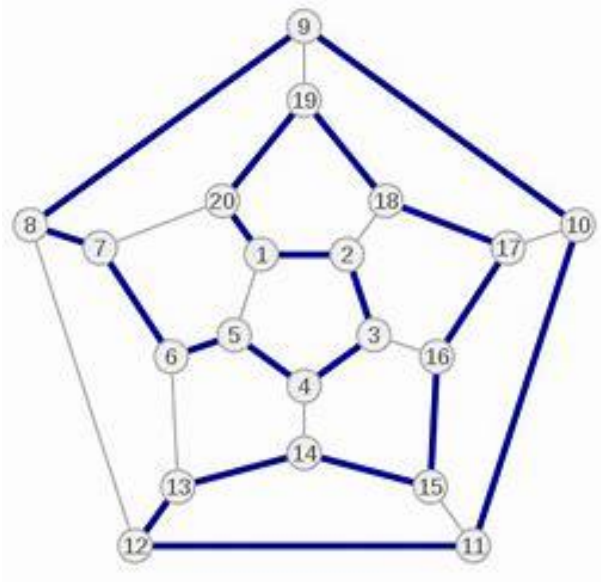
Graph-Isomorphism

- If two graphs are identical except for the names of the points, they are called **isomorphic**.
- Peggy knows the Isomorphism between two big Graphs G_1 and G_2
- (**Commitment**)Peggy generates graphs H_1, H_2, ….H_n, which are isomorphic to G_1 and G_2
- Victor asks Peggy to show isomorphism of H_i with either G_1 or G_2, **but not both**
- Why Zero-knowledge?
  - **NP problem!**



Hamiltonian Cycles

- A circular, continuous path along the lines of a graph that passes through each point exactly once
- (1) Peggy randomly permutes G to make a new graph, H. She then encrypts H to get H′.
- (2) Peggy gives Victor a copy of H′.
- (3) Victor asks Peggy either to:
  - (a) prove to him that H′ is an encryption of an isomorphic copy of G, or
  - (b) show him a Hamiltonian cycle for H.



- (4) Peggy complies. She either:
  - (a) proves that H′ is an encryption of an isomorphic copy of, or
  - (b) shows a Hamiltonian cycle for H.
- (5) Peggy and Victor repeat steps (1) through (4) n times.

Zero Knowledge Proof Properties

- **Completeness**: If the Prover is honest, then she will eventually convince the Verifier.
- **Soundness:** The Prover can only convince the Verifier if the statement is true.
- **Zero-knowledge(ness):** *The Verifier learns no information beyond the fact that the statement is true.*

Zero Knowledge Proof Fiat - Shamir protocol

- $n = pq$, $v=s^2 \bmod (n-1)$
- Prove knowledge of s given v.
- A chooses r and sends $x = r^2 \bmod n$ to B.
- B sends e = 0 or 1 to A
- A computes $y = r*s^e \bmod n$ and sends to B.
- B checks that $y^2 = x*v^e \bmod p$.
- Modular Quadratic Root

Zero Knowledge Proof of secret exponent

- Given a value **y**, a large prime **p** and a generator **g**, she wants to prove that she knows a value **x** such that g^x mod p=y, without revealing x.
- in each round, Peggy generates a random number r, computes $C=g^r$ mod p and discloses this to Victor.
- Victor requests r or (x+r) mod p-1
  - Verify $C = g^r$ if r is received
  - Verify $C*y = g^{x+r}$ mod p

Parallel Zero-Knowledge Proofs

- *n* interactions to parallel execution?
- (1) Peggy uses her information and *n* random numbers to transform the hard problem into *n* different isomorphic problems. She then uses her information and the random numbers to solve the *n* new hard problems.
- (2) Peggy commits to the solution of the *n* new hard problems.
- (3) Peggy reveals to Victor the *n* new hard problems.
- (4) For each of the *n* new hard problems, Victor asks Peggy either to:
  - (a) prove to him that the old and new problems are isomorphic, or
  - (b) open the solution she committed to in step (2) and prove that it is a solution to the new problem.
- (5) Peggy complies for each of the n new hard problems.
- Can we replace (4) with non-interactive operations?

Non-Interactive ZK Proof

- Uncertainty about "*which question would be asked*" is the backbone of ZKPS
- Interaction from Victor provides the **random** choice, or the element of uncertainty
- Use one-way (unpredictable) function as Victor surrogate
- Number of rounds must be large enough
  - (3) Peggy uses all of these commitments together as a single input to a one-way hash function. She then saves the first *n* bits of the output of this one-way hash function.
  - (4) Peggy takes the n bits generated in step (3). For each *i*th new hard problem in turn, she takes the *i*th bit of those n bits and:
    - (a) if it is a 0, she proves that the old and new problems are isomorphic, or
    - (b) if it is a 1, she opens the solution she committed to in step (2) and proves that it is a solution to the new problem.

Zerocoin: A more relevant example (IEEE S&P 2013)

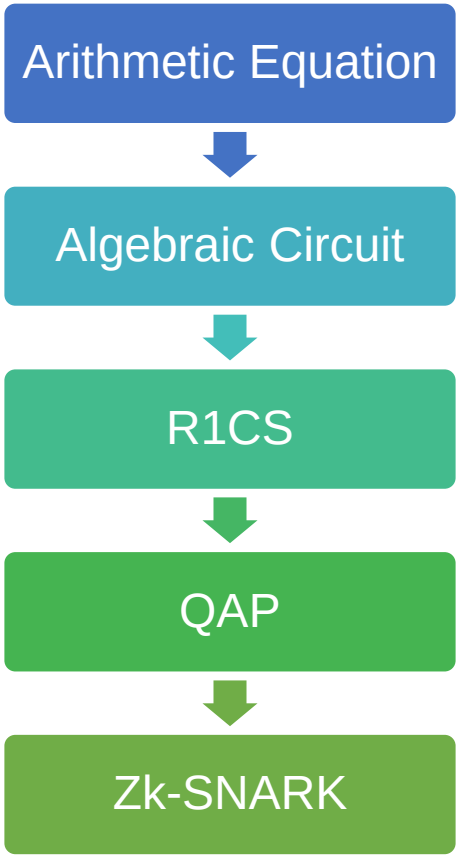
- Building blocks:
  - One-way Accumulator: Proving one value in a set
  - Non-interactive Zero Knowledge Proof
- Overview of Zerocoin
  - **Mint**: convert a Bitcoin into Zerocoin by $c=g^S h^r \bmod p$, where S, r is kept secretly.
  - **Spend**: from Zerocoin to Bitcoin. Providing a zero knowledge proof (π, S) of r for Zerocoin c.
  - Note: double spending is prevented by S, whose reuse can be immediately detected.
- Problems:
  - A decentralized mix
  - Amount exposed
  - ZKP: 45kB and 450ms for verification

Outline

Crypto Basics → Monero and RingCT → Zero Knowledge Proof → ZeroCash and ZK-SNARK

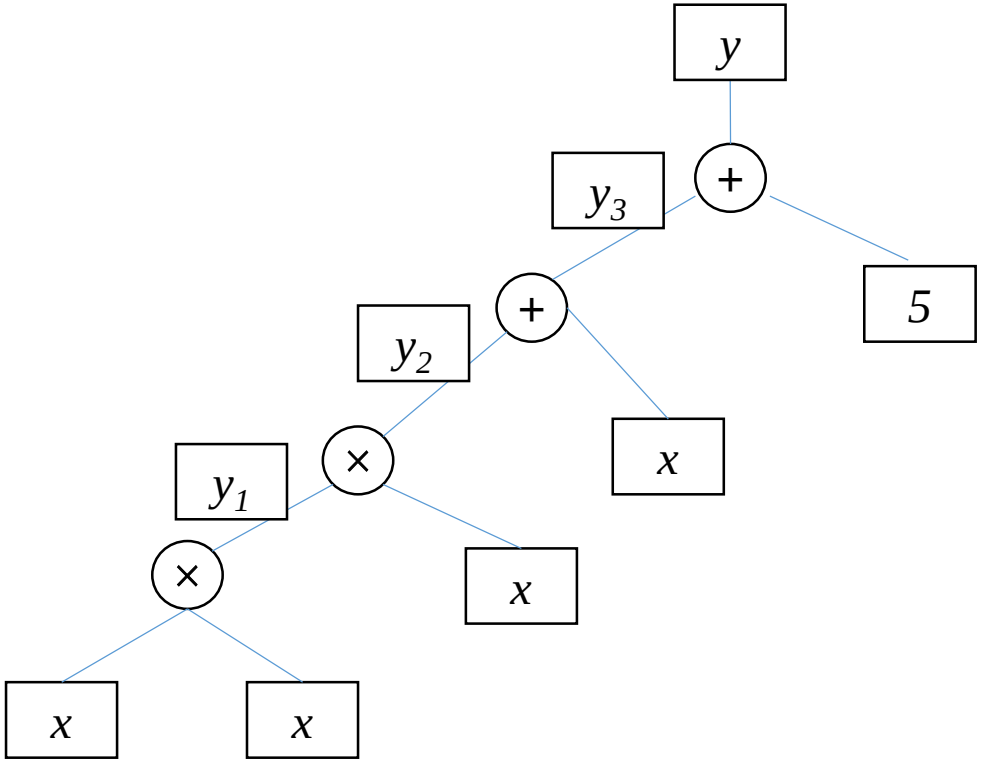
zk-SNARK

- Zero knowledge
- Succinct
- Non-interactive
- **Argument**
  - Computationally sound proof v.s. perfect soundness
  - Soundness: it is impossible to "prove" a false assertion
- Of Knowledge



zk-SNARK: A Simple Example

- Arithmetic equation: $x^3 + x + 5 = y$
- Proof: know an x satisfying the equation
- Step 1: from equation to arithmetic circuit



- $x^2 = y_1$
- $y_1 * x = y_2$
- $y_2 + x = y3$
- $y_3 + 5 = y$

zk-SNARK: A Simple Example (2)

- From Circuit to R1CS
  - $x^2 = y_1$
  - $y_1 * x = y_2$
  - $y_2 + x = y_3$
  - $y_3 + 5 = y$

- $S = [1, x, y, y_1, y_2, y_3]$
- $x = [0, 1, 0, 0, 0, 0] \cdot S$
- $y = [0, 0, 1, 0, 0, 0] \cdot S$
- $y_1 = [0, 0, 0, 1, 0, 0] \cdot S$
- $y_2 = [0, 0, 0, 0, 1, 0] \cdot S$
- $y_3 = [0, 0, 0, 0, 0, 1] \cdot S$

- R1CS: rank-1 constraint system ***(a, b, c)***
- A solution s satisfies R1CS if:

$$\boldsymbol{s \cdot a * s \cdot b - s \cdot c = 0}$$

- $a = [0, 1, 0, 0, 0, 0]$
- $b = [0, 1, 0, 0, 0, 0]$
- $c = [0, 0, 0, 1, 0, 0]$

- $a = [0, 0, 0, 1, 0, 0]$
- $b = [0, 1, 0, 0, 0, 0]$
- $c = [0, 0, 0, 0, 1, 0]$

- $a = [0, 1, 0, 0, 1, 0]$
- $b = [1, 0, 0, 0, 0, 0]$
- $c = [0, 0, 0, 0, 0, 1]$

- $a = [5, 0, 0, 0, 0, 1]$
- $b = [1, 0, 0, 0, 0, 0]$
- $c = [0, 0, 1, 0, 0, 0]$

zk-SNARK: A Simple Example (3)

- The solution S should satisfy all the four constraints
- Can satisfy them in one shot? **Quadratic Arithmetic Program (QAP)**

- $a = [0, 1, 0, 0, 0, 0]$
- $b = [0, 1, 0, 0, 0, 0]$
- $c = [0, 0, 0, 1, 0, 0]$

- $a = [0, 0, 0, 1, 0, 0]$
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- $c = [0, 0, 0, 0, 1, 0]$

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- $c = [0, 0, 0, 0, 0, 1]$

- $a = [5, 0, 0, 0, 0, 1]$
- $b = [1, 0, 0, 0, 0, 0]$
- $c = [0, 0, 1, 0, 0, 0]$

- A: $[0, 1, 0, 0, 0, 0]$
- $[0, 0, 0, 1, 0, 0]$
- $[0, 1, 0, 0, 1, 0]$
- $[5, 0, 0, 0, 0, 1]$
- B: $[0, 1, 0, 0, 0, 0]$
- $[0, 1, 0, 0, 0, 0]$
- $[1, 0, 0, 0, 0, 0]$
- $[1, 0, 0, 0, 0, 0]$
- C: $[0, 0, 0, 1, 0, 0]$
- $[0, 0, 0, 0, 1, 0]$
- $[0, 0, 0, 0, 0, 1]$
- $[0, 0, 1, 0, 0, 0]$

- $A(1) = [0, 1, 0, 0, 0, 0]$
- $A(2) = [0, 0, 0, 1, 0, 0]$
- $A(3) = [0, 1, 0, 0, 1, 0]$
- $A(4) = [5, 0, 0, 0, 0, 1]$
- $B(1) = [0, 1, 0, 0, 0, 0]$
- $B(2) = [0, 1, 0, 0, 0, 0]$
- $B(3) = [1, 0, 0, 0, 0, 0]$
- $B(4) = [1, 0, 0, 0, 0, 0]$
- $C(1) = [0, 0, 0, 1, 0, 0]$
- $C(2) = [0, 0, 0, 0, 1, 0]$
- $C(3) = [0, 0, 0, 0, 0, 1]$
- $C(4) = [0, 0, 1, 0, 0, 0]$

zk-SNARK: A Simple Example (3)

- The solution S should satisfy all the four constraints
- Can satisfy them in one shot? **Quadratic Arithmetic Program (QAP)**

- $a = [0, 1, 0, 0, 0, 0]$
- $b = [0, 1, 0, 0, 0, 0]$
- $c = [0, 0, 0, 1, 0, 0]$

- $A: [0, 1, 0, 0, 0, 0]$
- $[0, 0, 0, 1, 0, 0]$
- $[0, 1, 0, 0, 1, 0]$
- $[5, 0, 0, 0, 0, 1]$

- $A(1) = [0, 1, 0, 0, 0, 0]$
- $A(2) = [0, 0, 0, 1, 0, 0]$
- $A(3) = [0, 1, 0, 0, 1, 0]$
- $A(4) = [5, 0, 0, 0, 0, 1]$

- $A(x) = [A_1(x), A_2(x), A_3(x), A_4(x), A_5(x), A_6(x)]$
- $B(x) = [B_1(x), B_2(x), B_3(x), B_4(x), B_5(x), B_6(x)]$
- $C(x) = [C_1(x), C_2(x), C_3(x), C_4(x), C_5(x), C_6(x)]$

$B(1) = [0, 1, 0, 0, 0, 0]$

$B(2) = [0, 1, 0, 0, 0, 0]$

$B(3) = [1, 0, 0, 0, 0, 0]$

$B(4) = [1, 0, 0, 0, 0, 0]$

- $b = [1, 0, 0, 0, 0, 0]$
- $c = [0, 0, 0, 0, 0, 1]$

- $[1, 0, 0, 0, 0, 0]$
- $C: [0, 0, 0, 1, 0, 0]$
- $[0, 0, 0, 0, 1, 0]$
- $[0, 0, 0, 0, 0, 1]$
- $[0, 0, 1, 0, 0, 0]$

- $C(1) = [0, 0, 0, 1, 0, 0]$
- $C(2) = [0, 0, 0, 0, 1, 0]$
- $C(3) = [0, 0, 0, 0, 0, 1]$
- $C(4) = [0, 0, 1, 0, 0, 0]$

- $a = [5, 0, 0, 0, 0, 1]$
- $b = [1, 0, 0, 0, 0, 0]$
- $c = [0, 0, 1, 0, 0, 0]$

zk-SNARK: A Simple Example (4)

- The final QAP by Lagrange interpolation

- *A polynomials*
- *[-5.0, 9.166, -5.0, 0.833]* $= -5 + 9.166x - 5x^2 + 0.833x^3$
- *[8.0, -11.333, 5.0, -0.666]* $= 8 - 11.333x + 5x^2 - 0.666x^3$
- *[0.0, 0.0, 0.0, 0.0]*
- *[-6.0, 9.5, -4.0, 0.5]*
- *[4.0, -7.0, 3.5, -0.5]*
- *[-1.0, 1.833, -1.0, 0.166]*

- *B polynomials*
- *[3.0, -5.166, 2.5, -0.333]*
- *[-2.0, 5.166, -2.5, 0.333]*
- *[0.0, 0.0, 0.0, 0.0]*
- *[0.0, 0.0, 0.0, 0.0]*
- *[0.0, 0.0, 0.0, 0.0]*
- *[0.0, 0.0, 0.0, 0.0]*

- *C polynomials*
- *[0.0, 0.0, 0.0, 0.0]*
- *[0.0, 0.0, 0.0, 0.0]*
- *[-1.0, 1.833, -1.0, 0.166]*
- *[4.0, -4.333, 1.5, -0.166]*
- *[-6.0, 9.5, -4.0, 0.5]*
- *[4.0, -7.0, 3.5, -0.5]*

- A(1) =*[0, 1, 0, 0, 0, 0]*
- A(2) =*[0, 0, 0, 1, 0, 0]*
- A(3) =*[0, 1, 0, 0, 1, 0]*
- A(4) =*[5, 0, 0, 0, 0, 1]*
- *B(1) =[0, 1, 0, 0, 0, 0]*
- *B(2) =[0, 1, 0, 0, 0, 0]*
- *B(3) =[1, 0, 0, 0, 0, 0]*
- *B(4) =[1, 0, 0, 0, 0, 0]*
- *C*(1)=*[0, 0, 0, 1, 0, 0]*
- *C*(2)=*[0, 0, 0, 0, 1, 0]*
- *C*(3)=*[0, 0, 0, 0, 0, 1]*
- *C*(4)=*[0, 0, 1, 0, 0, 0]*

zk-SNARK: A Simple Example (5)

- So where are we?
- We want to prove we know a secret x satisfying an equation: $x^3 + x + 5 = y$
- We convert it into a circuit
- The circuit is converted into a set of R1CS (a, b, c)
- R1CS is then transformed to QAP ($A(x)$, $B(x)$, $C(x)$)
- Now a solution s should satisfy:
  $s \cdot A(x) * s \cdot B(x) - s \cdot C(x) = H(x) * (x-1)(x-2)(x-3)(x-4) = H(x)*Z(x)$
  - $s \cdot A(1) * s \cdot B(1) - s \cdot C(1) = 0$
  - $s \cdot A(2) * s \cdot B(2) - s \cdot C(2) = 0$
  - $s \cdot A(3) * s \cdot B(3) - s \cdot C(3) = 0$
  - $s \cdot A(4) * s \cdot B(4) - s \cdot C(4) = 0$
  - When $x=1,2,3,4$, both sides evaluate to 0

zk-SNARK: A Simple Example (6)

- Solution $s = [1, x, y, y_1, y_2, y_3] = [1, 3, 35, 9, 27, 30]$
- $A \cdot s = [43.0, -73.333, 38.5, -5.166]$
- $B \cdot s = [-3.0, 10.333, -5.0, 0.666]$
- $C \cdot s = [-41.0, 71.666, -24.5, 2.833]$

- $t = A \cdot s * B \cdot s - C \cdot s =$
- $[-88.0, 592.666, -1063.777, 805.833, -294.777, 51.5, -3.444]$
- $Z=(x-1)(x-2)(x-3)(x-4) \rightarrow [24, -50, 35, -10, 1]$
- $h = t / Z = [-3.666, 17.055, -3.444]$

zk-SNARK: A Simple Example (7)

- Note:
  - $A(x)$, $B(x)$, $C(x)$ are determined by the equation we want to prove for some x
  - $s \cdot A(x) * s \cdot B(x) != s^2 * A(x) * B(x)$
- $s \cdot A(x) * s \cdot B(x) - s \cdot C(x) = H(x) * (x-1)(x-2)(x-3)(x-4)$
  - *$A(x)$, $B(x)$, $C(x)$ are known*
  - *$H(x)$ is determined by s*
- Prove equality at some secret point α (no one knows it!)
- $s \cdot A(\alpha) * s \cdot B(\alpha) - s \cdot C(\alpha) = H(\alpha) * (\alpha -1)(\alpha -2)(\alpha -3)(\alpha -4) = H(\alpha)*Z(\alpha)$

zk-SNARK: why it works?

- Constraints enforced by the polynomials
  - Polynomials (of R1CS) evaluated at different points (1,2,3,4) corresponds to different constraint
  - If a solution satisfies R1CS specified by the polynomials, then the solution must satisfy all constraints specified by the R1CS
- Verify equality at a secret point α instead of polynomials
  - *If no one knows* α*, and* $s \cdot A(\alpha) * s \cdot B(\alpha) - s \cdot C(\alpha) = H(\alpha) * Z(\alpha)$*, then* $s \cdot A(x) * s \cdot B(x) - s \cdot C(x) = H(x) * Z(x)$ *for any* x.
  - *Evaluation of* $A(\alpha)$*,* $B(\alpha)$*,* $C(\alpha)$ *and* $H(\alpha)$ *at secret* α
- *Need to protect* s *and* α

zk-SNARK: A Simple Example (8)

- Prover knows a solution s, wanting to prove it satisfies the constraints specified by QAP
  - E.g. a Zerocash transaction should satisfy constraints on input values, output values.
- $s \cdot A(x) * s \cdot B(x) - s \cdot C(x) = H(x) * (x-1)(x-2)(x-3)(x-4)$
- Use discrete log to hide s
- Recall: $y = g^x \bmod p$, infeasible to obtain x with y and p, where p is a large prime.
- That is: $g^{s \cdot A(x) * s \cdot B(x) - s \cdot C(x)} = g^{H(x) * (x-1)(x-2)(x-3)(x-4)}$?
- Not that simple!

zk-SNARK: Evaluation at a secret point α

- $g^{s \cdot A(\alpha) * s \cdot B(\alpha) - s \cdot C(\alpha)} = g^{H(\alpha) * (\alpha -1)(\alpha -2)(\alpha -3)(\alpha -4)}$
- $g^{P(\alpha)} = g^{a_0 + a_1\alpha + \cdots + a_n\alpha^n} = g^{a_0}(g^{\alpha})^{a_1} \ldots (g^{\alpha^n})^{a_n}$
  - Recall $A(\alpha) = [A_1(\alpha), A_2(\alpha), \ldots, A_6(\alpha)]$
  - Easy to compute $g^{A_i(\alpha)}$ as $A_i(\alpha)$ is fixed for a circuit
- $s = [1, x, y, y_1, y_2, y_3] = [c_1, c_2, c_3, c_4, c_5, c_6]$, a given solution.
- $g^{s \cdot A(\alpha)} = g^{\sum c_i A_i(\alpha)} = \prod (g^{A_i(\alpha)})^{c_i}$ without leaking c_i
- Same way to compute $g^{s \cdot B(\alpha)}, g^{s \cdot C(\alpha)}, g^{H(\alpha)}, g^{(\alpha-1)(\alpha-2)(\alpha-3)(\alpha-4)}$
  - Public key: $g^{A_i(\alpha)}$, $g^{B_i(\alpha)}$, $g^{C_i(\alpha)}$, g^{α},…, g^{α^k}
  - H(α) depends on s, the given inputs and outputs

zk-SNARK: A Summary

- Step 1: Given a set of arithmetic equations to be verified:
  - $H(x) = y$
  - $x + y = z$
- Step 2: Convert equations to R1CS
- Step 3: Convert R1CS to QAP by Lagrange interpolation
- Step 4: Convert to zk-SNARK

Zerocash: Improvement Over Zerocoin (IEEE S&P 2014)

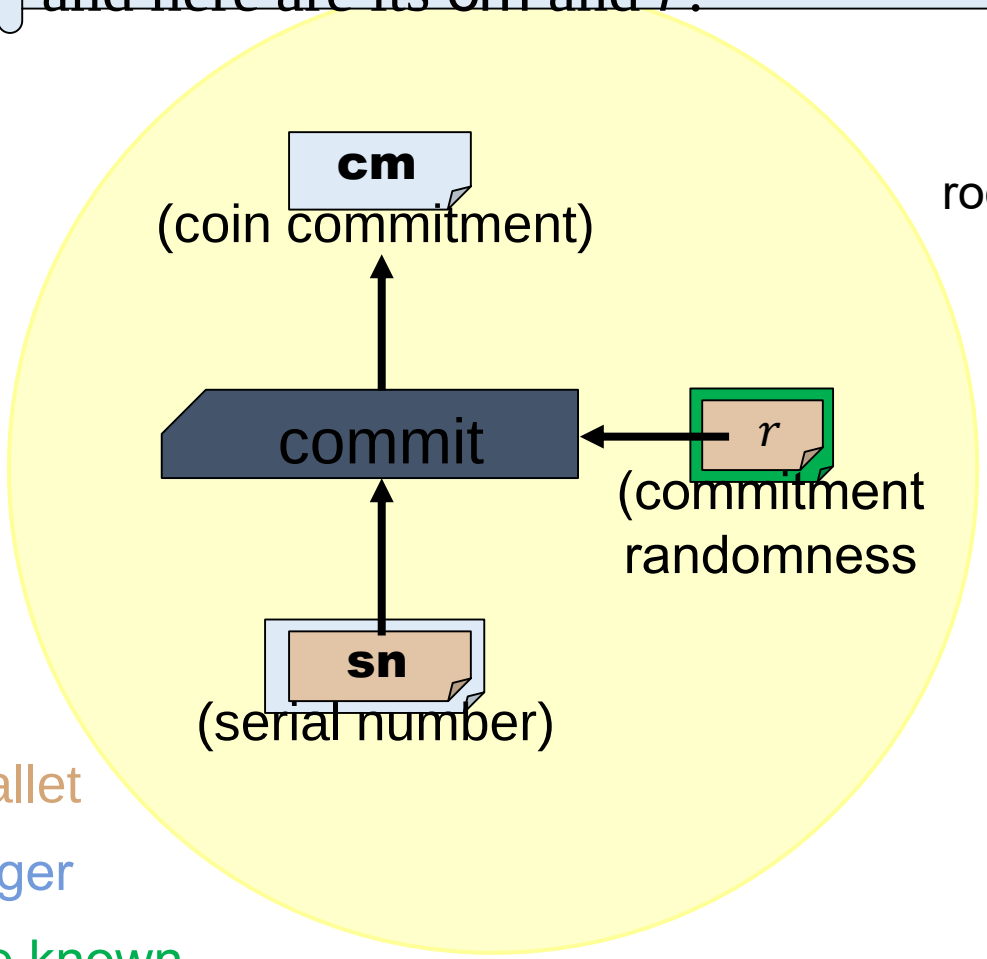
- Zk-SNARKs: zero-knowledge Succinct Non-interactive Arguments of Knowledge.
- Overview:
  - **Mint** coins: from Bitcoin to Zerocash.
  - **Pour**: from two inputs to two new addresses, where zk-SNARK is used.

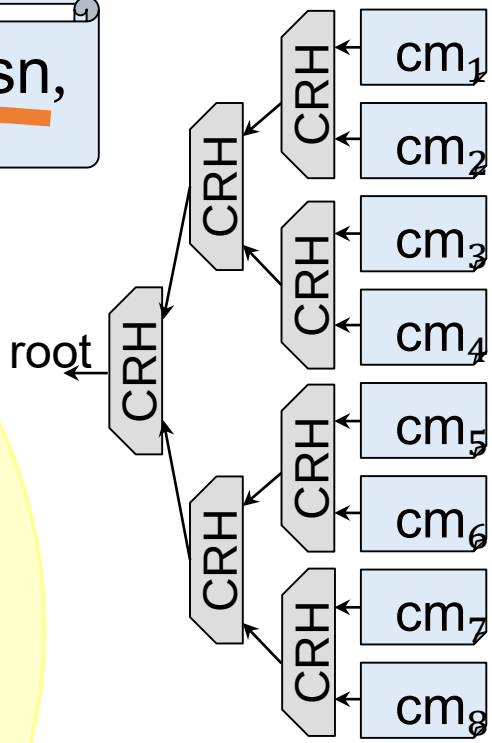
Intel Core i7-4770 @ 3.40GHz with 16GB of RAM (1 thread)		
Setup	Time	5 min 17 s
	pp	896 MiB
CreateAddress	Time	326.0 ms
	$\mathsf{addr}_{\mathsf{pk}}$	343 B
	$\mathsf{addr}_{\mathsf{sk}}$	319 B
Mint	Time	23 µs
	Coin $\mathbf{c}$	463 B
	$\mathsf{tx}_{\mathsf{Mint}}$	72 B
Pour	Time	2 min 2.01 s
	$\mathsf{tx}_{\mathsf{Pour}}$	996 B[16]
VerifyTransaction	mint	8.3 µs
	pour (excludes L scan)	5.7 ms
Receive	Time (per pour tx)	1.6 ms

Basic anonymous e-cash [Sander Ta-Shma 1999]

Minting: I hereby spend 1 BTC to create cm

Spending: ~~I'm using up a coin with (unique)~~ sn, and here are its cm and r.





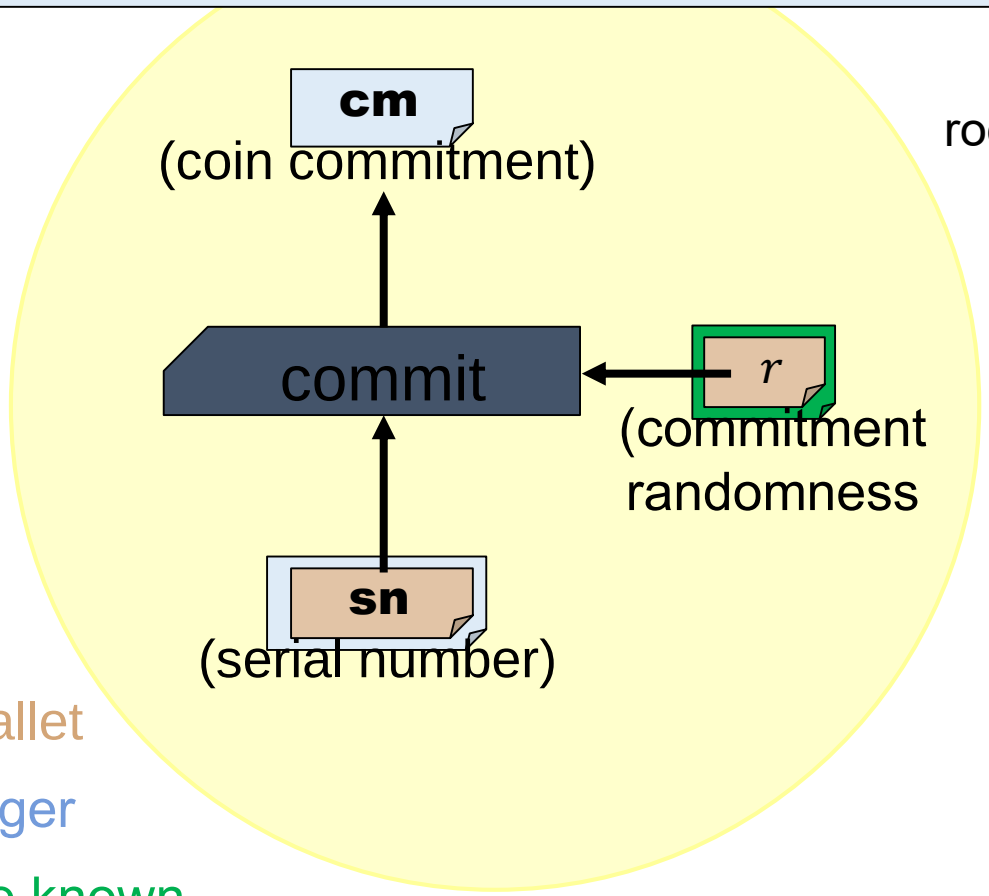
Legend:

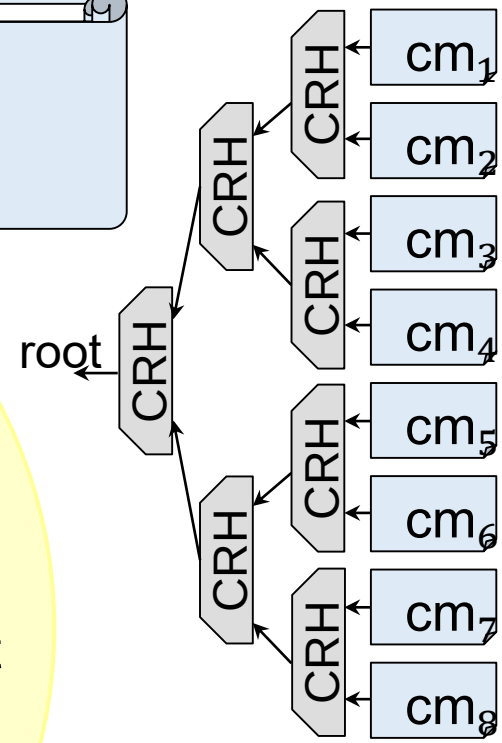
- In private wallet
- In public ledger
- Proved to be known

Basic anonymous e-cash [Sander Ta-Shma 1999]

Minting: I hereby spend 1 BTC to create cm

Spending: I'm using up a coin with (unique) sn, and I know r, and a cm in the tree with root, that match sn.





Legend:

- In private wallet
- In public ledger
- Proved to be known

Basic anonymous e-cash – requisite proofs

Spending:

I'm using up a coin with (unique) sn,
and I know a cm in the tree, and r,
that match sn.

Requires:

zero **k**nowledge

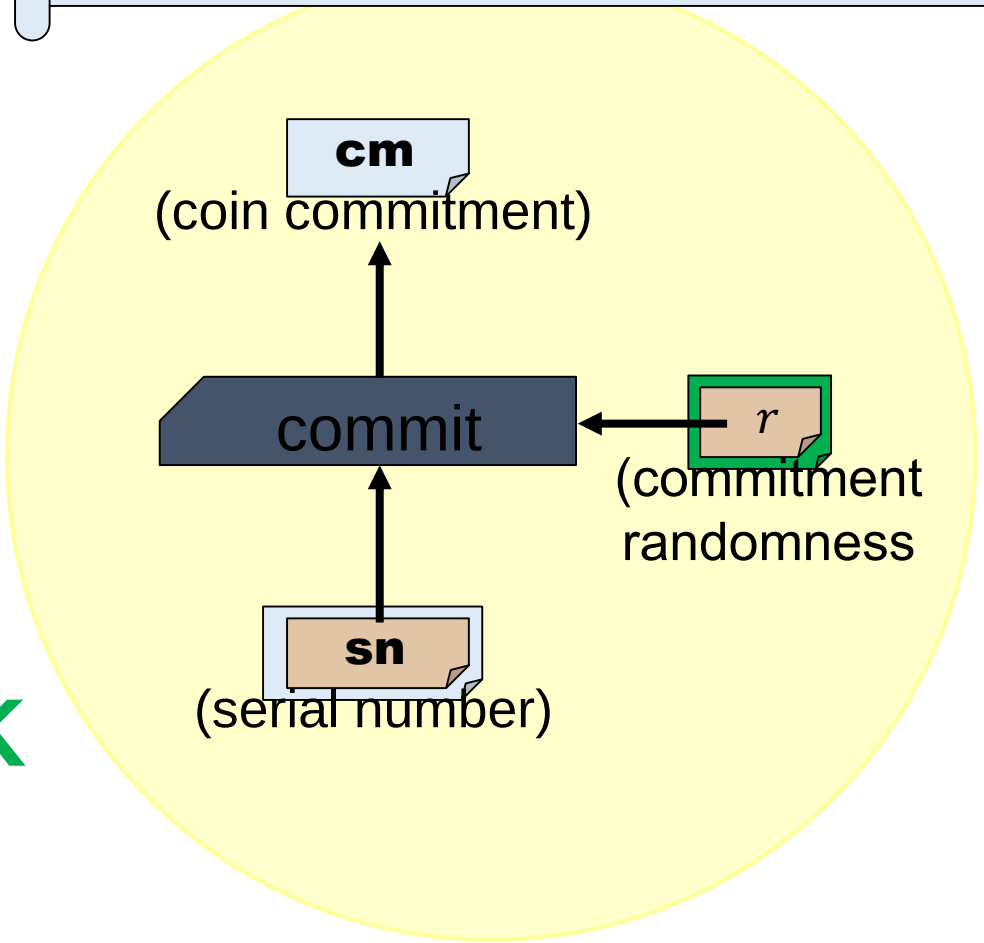
succinct

noninteractive

argument

proof

of **k**nowledge



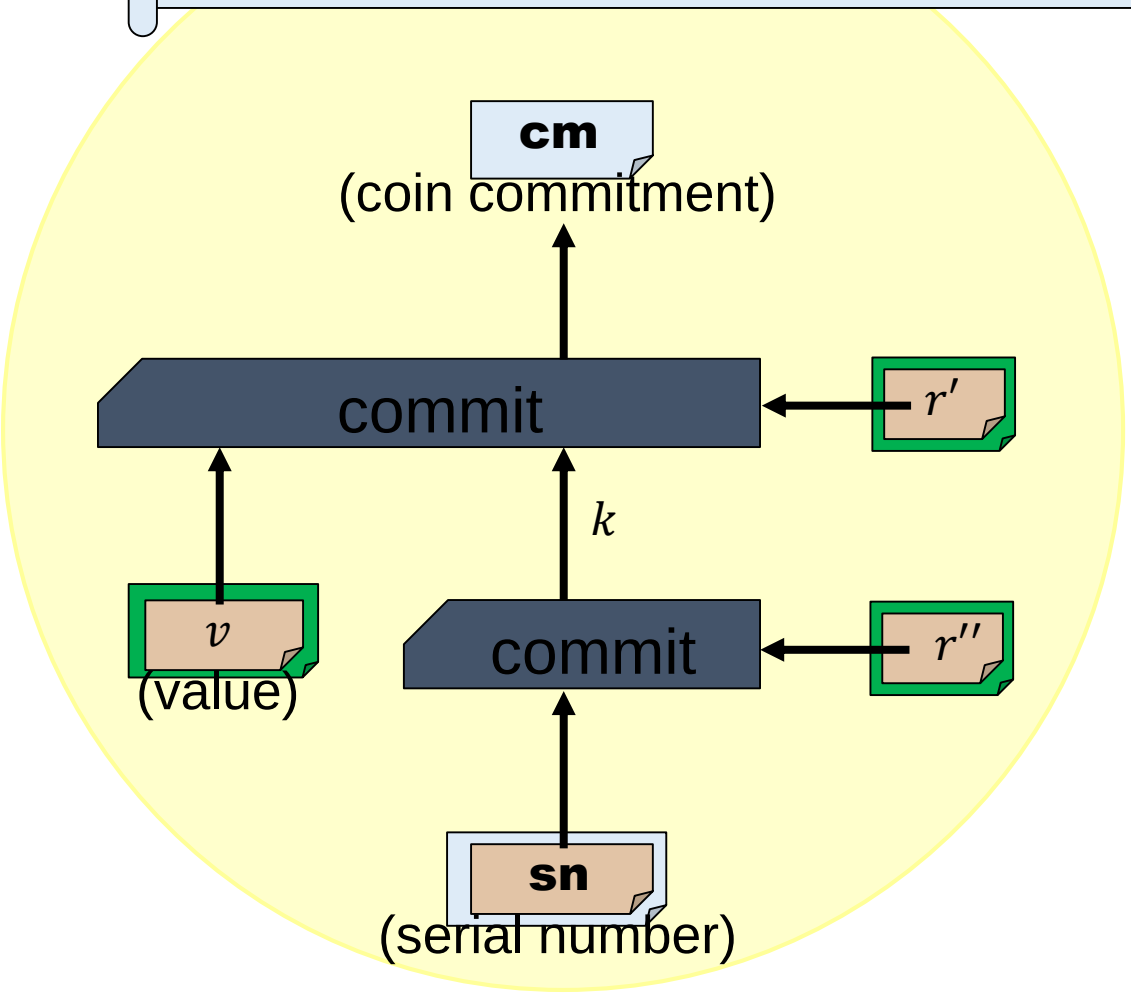
zkSNARK

Adding variable denomination

Minting: I hereby spend v BTC to create cm, and here is k, r' to prove consistency

Spending: I'm using up a coin with value v (unique) sn, and I know r', r'' that are consistent with cm.

zkSNARK

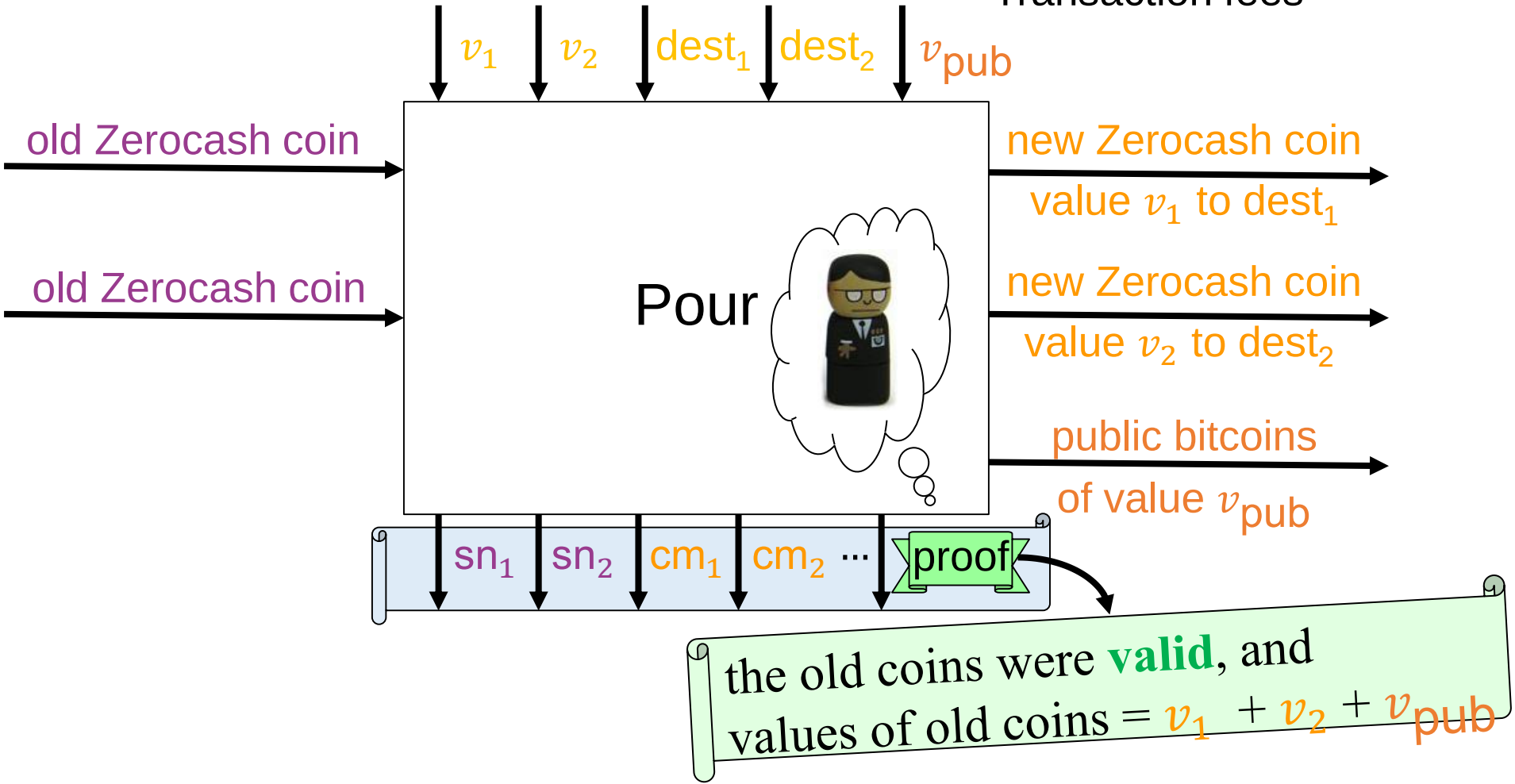


Pouring Zerocash coins

Simplified

Single transaction type capturing:

- Sending payments
- Making change
- Exchanging into bitcoins
- Transaction fees



Pouring Zerocash coins

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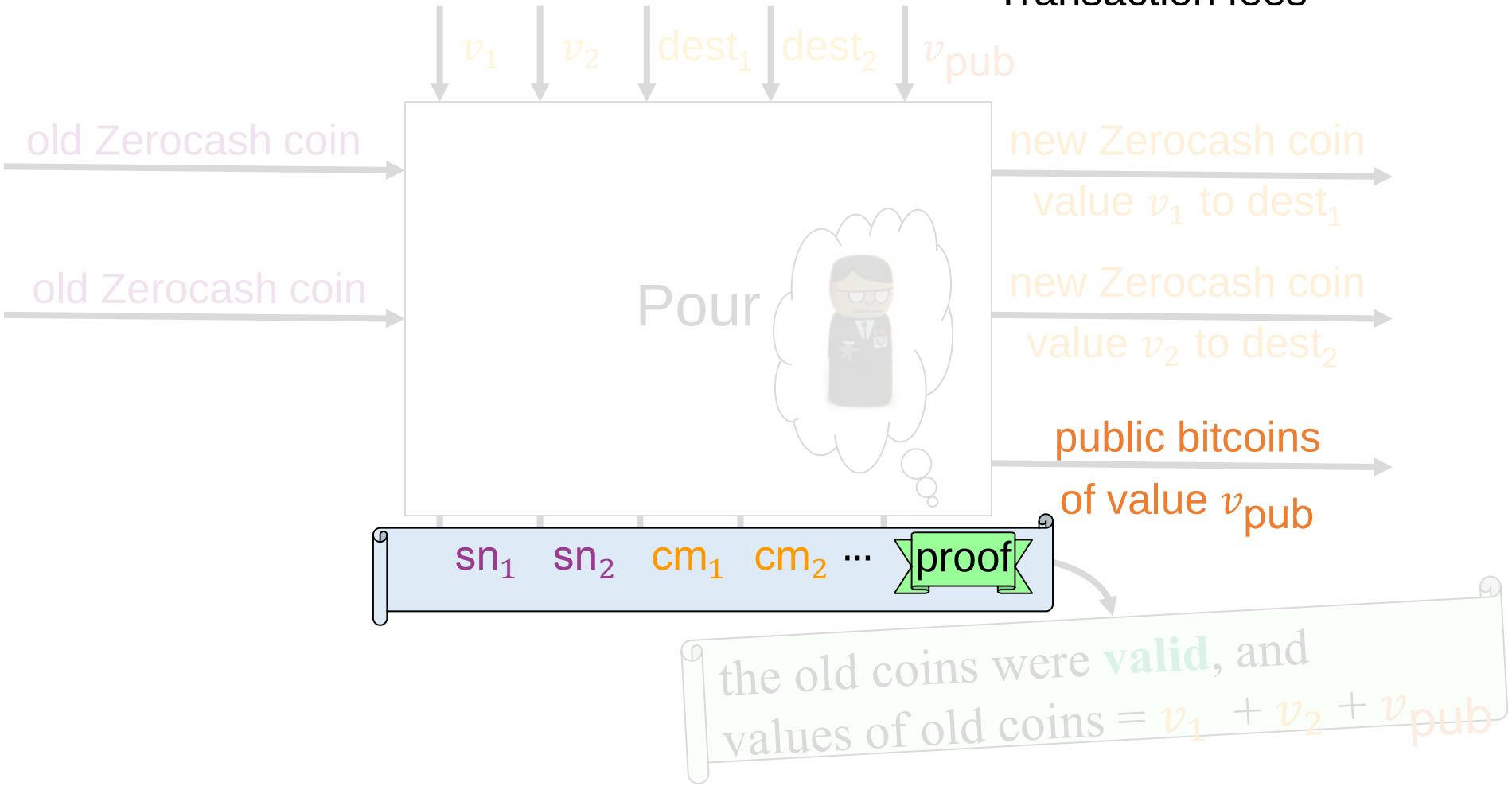

What zk-SNARK can do?

- Inputs equal to outputs
- Hash value matches without disclosing preimage
- Inputs are valid and not double spending

$$\mathsf{tx}_{\mathsf{Pour}} = (\mathsf{rt}, \mathsf{sn}_1^{\mathsf{old}}, \mathsf{sn}_2^{\mathsf{old}}, \mathsf{cm}_1^{\mathsf{new}}, \mathsf{cm}_2^{\mathsf{new}}, v_{\mathsf{pub}}, \mathsf{info}, *)$$

(a) The coin commitment $\mathsf{cm}_i^{\mathsf{old}}$ of $\mathbf{c}_i^{\mathsf{old}}$ appears on the ledger, i.e., path_i is a valid authentication path for leaf $\mathsf{cm}_i^{\mathsf{old}}$ with respect to root rt, in a CRH-based Merkle tree.

(b) The address secret key $a_{\mathsf{sk},i}^{\mathsf{old}}$ matches the address public key of $\mathbf{c}_i^{\mathsf{old}}$, i.e., $a_{\mathsf{pk},i}^{\mathsf{old}} = \mathsf{PRF}_{a_{\mathsf{sk},i}^{\mathsf{old}}}^{\mathsf{addr}}(0)$.

(c) The serial number $\mathsf{sn}_i^{\mathsf{old}}$ of $\mathbf{c}_i^{\mathsf{old}}$ is computed correctly, i.e., $\mathsf{sn}_i^{\mathsf{old}} = \mathsf{PRF}_{a_{\mathsf{sk},i}^{\mathsf{old}}}^{\mathsf{sn}}(\rho_i^{\mathsf{old}})$.

(d) The coin $\mathbf{c}_i^{\mathsf{old}}$ is well-formed, i.e., $\mathsf{cm}_i^{\mathsf{old}} = \mathsf{COMM}_{s_i^{\mathsf{old}}}(\mathsf{COMM}_{r_i^{\mathsf{old}}}(a_{\mathsf{pk},i}^{\mathsf{old}} \| \rho_i^{\mathsf{old}}) \| v_i^{\mathsf{old}})$.

(e) The coin $\mathbf{c}_i^{\mathsf{new}}$ is well-formed, i.e., $\mathsf{cm}_i^{\mathsf{new}} = \mathsf{COMM}_{s_i^{\mathsf{new}}}(\mathsf{COMM}_{r_i^{\mathsf{new}}}(a_{\mathsf{pk},i}^{\mathsf{new}} \| \rho_i^{\mathsf{new}}) \| v_i^{\mathsf{new}})$.

(f) The address secret key $a_{\mathsf{sk},i}^{\mathsf{old}}$ ties h_{Sig} to h_i, i.e., $h_i = \mathsf{PRF}_{a_{\mathsf{sk},i}^{\mathsf{old}}}^{\mathsf{pk}}(i \| h_{\mathsf{Sig}})$.

2. Balance is preserved: $v_1^{\mathsf{new}} + v_2^{\mathsf{new}} + v_{\mathsf{pub}} = v_1^{\mathsf{old}} + v_2^{\mathsf{old}}$ (with $v_1^{\mathsf{old}}, v_2^{\mathsf{old}} \geq 0$ and $v_1^{\mathsf{old}} + v_2^{\mathsf{old}} \leq v_{\mathsf{max}}$).

Example of a Zerocash Pour transaction

root	1c4ac4a110e863deeca050dc5e5153f2b7010af9
sn_1	a365e7006565f14342df9096b46cc7f1d2b9949367180fdd8de4090eee30bfdc
sn_2	6937031dce13facdebe79e8e2712ffad2e980c911e4cec8ca9b25fc88df73b52
cm_1	a4d015440f9cfae0c3ca3a38cf04058262d74b60cb14ecd6063e047694580103
cm_2	2ca1f833b63ac827ba6ae69b53edc855e66e2c2d0a24f8ed5b04fa50d42dc772
v_pub	0000000000000042
pubkeyHash info	8f9a43f0fe28bef052ec209724bb0e502ffb5427
SigPK	2dd489d97daa8ceb006cb6049e1699b16a6d108d43
Sig	f1d2d2f924e986ac86fdf7b36c94bcdf32beec15a38359c82f32dbb3342cb4bedcb78ce116bac69e
MAC_1	b8a5917eca1587a970bc9e3ec5e395240ceb1ef700276ec0fa92d1835cb7f629
MAC_2	ade6218b3a17d609936ec6894b7b2bb446f12698d4bcafa85fcbf39fb546603a
ciphertext_1	048070fe125bdaf93ae6a7c08b65adbb2a438468d7243c74e80abc5b74dfe3524a987a2e3ed075d54ae7a53866973eaa5070c4e0895 4ff5d80caae214ce572f42dc6676f0e59d5b1ed68ad33b0c73cf9eac671d8f0126d86b667b319d255d7002d0a02d82efc47fd8fd648 057fa823a25dd3f52e86ed65ce229db56816e646967baf4d2303af7fe09d24b8e30277336cb7d8c81d3c786f1547fe0d00c029b63bd 9272aad87b3f1a2b667fa575e
ciphertext_2	0493110814319b0b5cabb9a9225062354987c8b8f604d96985ca52c71a77055b4979a50099cefc5a359bdf0411983388fa5de840a0d 64816f1d9f38641d217986af98176f420caf19a2dc18c79abcf14b9d78624e80ac272063e6b6f78bc42c6ee01edfbcddbeb60eba586 eaecd6cb017069c8be2ebe8ae8a2fa5e0f6780a4e2466d72bc3243e873820b2d2e4b954e9216b566c140de79351abf47254d122a35f 17f840156bd7b1feb942729dc
zkSNARKproof	a4c3cad6e02eec51dc8a37ebc51885cf86c5da04bb1c1c0bf3ed97b778277fb8adceb240c40a0cc3f2854ce3df1eafdcefccc532bc5afaefefe9d3975726f2ca829228 6ca8dd4f8da21b3f98c61fac2a13f0b82544855b1c4ce7a0c9e57592ee1d233d43a2e76b9bdeb5a365947896f117002b095f7058bdf611e20b6c2087618c58208e3 658cfcc00846413f8f355139d0180ac11182095cdee6d9432287699e76ed7832a5fc5dc30874ff0982d9658b8e7c51523e0fa1a5b649e3df2c9ff58dc05dac7563741 298025f806dfbe9cfe5c8c40d1bf4e87dacb11467b9e6154fb9623d3fba9e7c8ad17f08b17992715dfd431c9451e0b59d7dc506dad84aef98475d4be530eb501925 dfd22981a2970a3799523b99a98e50d00eaab5306c10be5

~1KB total. Less without direct payments and public outputs.

Tutorial Examples

- libsnark lightning circuit
  - (source) https://github.com/wanzhiguo/lightning_circuit
- zksnark-toy
  - (source) https://github.com/wanzhiguo/zksnark-toy
- libsnark tutorial (originally by HowardWu from UC Berkerly)
  - https://github.com/SadPencil/libsnark-tutorial
  - git clone http://github.com/SadPencil/libsnark-tutorial
- Checkout Zerocash's zkSNARK
- SCIPR-lab/libsnark on github.com

Lightning Circuit

- Given H_1, H_2, X, prove one knows (R_1, R_2) *s.t.*
  - $H_1 = sha256(R_1)$ *and*
  - $H_2 = sha256(R_2)$ *and*
  - $R_1 = R_2$ *XOR* X
- (docker image)
  - make
  - ./test
- Implementation
  - Compose a gadget: l_gadget
  - Write test.cpp to test it

zksnark-toy

- Given a Merkle root rt, a leaf node leaf, a path from leaf to rt, prove one knows pre_leaf s.t.
  - leaf = sha256(pre_leaf)
  - leaf is valid w.r.t. rt and path
- (docker image)
  - chmod +x get-libsnark
  - ./get-libsnark
  - make
  - src/main
- Implementation
  - toy-gadget
  - main.cpp for test

Experiments

- Write simple gadgets based on examples
  - lightning_circuit
  - zksnark-toy
- Test gadgets with concrete instances
  - Compile your code
  - Run the code
- Your tasks:
  - H(x) = y
  - x + y = z
  - Any other interesting proof?

References

- Understanding Monero Cryptography, Privacy Part 2 -- Stealth Addresses. https://steemit.com/monero/@luigi1111/understanding-monero-cryptography-privacy-part-2-stealth-addresses
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- Monero explorer. Moneroexplorer.com
- Monero Trasaction Size. https://monero.stackexchange.com/questions/5664/size-requirements-for-different-pieces-of-a-monero-transaction
- **Explaining SNARKs. https://blog.z.cash/snark-explain1/**
- Quadratic Arithmetic Programs: from Zero to Hero. **https://medium.com/@VitalikButerin/quadratic-arithmetic-programs-from-zero-to-hero-f6d558cea649**
- **Explaining zkSNARK: https://blockchain.iethpay.com/zero-knowledge-zkSNARKs.html**
- **lightning_circuit: https://github.com/ebfull/lightning_circuit**
- **Libsnark**: Github.com/SCIPR-lab/libsnark
- *Pinocchio: Nearly Practical Verifiable Computation* , Bryan Parno, Craig Gentry, Jon Howell, Mariana Raykova, IEEE Symposium on Security and Privacy (Oakland) 2013

Thanks for your attention!

Questions?